Drying of Shrinking Bodies

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The traditional approach in studying the isothermal nonsteady diffusion in an infinite porous slab is based on Fick's second law:

$$\partial \rho_A/\partial \theta = D\partial^2 \rho_A/\partial x^2 \tag{1}$$

When shrinking or swelling is important, Fick's second law is given as

$$\partial\, u/\partial \theta = D_{\text{eff}}\partial^2 u/\partial x^2 \tag{2}$$

Sherwood (1929) proposed Eq. 2 to analyze the drying process in an infinite slab.

Assuming uniform initial moisture content, constant mass diffusivity, diffusion normal to the surface plane, evaporation occurring at the surface of the slab, and negligible surface resistance to vapor diffusion, the solution of Eq. 2 is, according to Luikov (1968)

$$(\bar{u} - u_i)/(u_o - u_i) = \sum_{n=1}^{\infty} 8/(\pi(2n-1))^2 \exp(-\pi^2/4(2n-1)^2 D\theta/R^2) \tag{3}$$

The flux of A across a B-fixed section is (Crank, 1956)

$$F_A = -D_A^B \partial C_A^B/\partial \xi_B \tag{4}$$

Performing a differential balance in the slab and taking into account Eq. 4

$$\partial C_A^B/\partial \theta = \partial/\partial \xi_B (D_A^B C_A^B/\partial \xi_B) \tag{5}$$

where D_A^B is given by Eqs. 11–28 in Crank (1956) as

$$D_A^B = D(\rho_B/\rho_s)^2 \tag{6}$$

and C_A^B is related to the moisture content, expressed on dry basis, by the equation

$$C_A^B = u\rho_s \tag{7}$$

Assuming unidirectional shrinking and volume additivity of A and B, the following equations can be obtained:

$$\rho_A/\rho_w + \rho_B/\rho_s = 1 \tag{8}$$

$$R_o = R_s(1 + \rho_s u_o/\rho_w) \tag{9}$$

$$R_i = R_s(1 + \rho_s u_i/\rho_w) \tag{10}$$

$$u = \rho_A/\rho_B \tag{11}$$

$$\rho_{Bo}/\rho_s = 1/(1 + \rho_s u_o/\rho_w) \tag{12}$$

$$\rho_{Bi}/\rho_s = 1/(1 + \rho_s u_i/\rho_w) \tag{13}$$

$$\bar{R}\rho_B = R_i\rho_{Bi} = R_o\rho_{Bo} = R_s\rho_s \tag{14}$$

By means of these equations, Eq. 5 can be put in the form

$$\partial u/\partial F_s = \partial/\partial_z(1/(1 + \rho_s u/\rho_w)^2 \partial u/\partial z) \tag{15}$$

$$\partial v/\partial F_o = \partial/\partial_z(1/v^2 \partial v/\partial z) \tag{16}$$

It can be observed that Eqs. 2 and 15 are clearly different. It is worth noting that Eq. 15 contains the equivalent to an explicit dependence of the diffusion coefficient with moisture content. It can be observed also that the dependent variable v in Eq. 16 has an initial value equal to unity, with boundary conditions given by

$$v_i = (1 + \rho_s u_i/\rho_w)/(1 + \rho_s u_o/\rho_w) = R_i/R_o \tag{17}$$

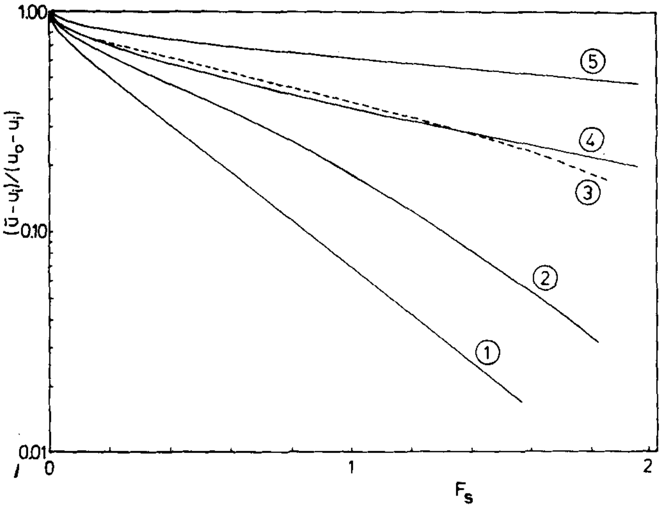

Figure 1. Dimensionless moisture content vs. Fourier number based on the dry half-thickness of the slab. Curve 1: Equation 3 (no shrinkage, R_s as characteristic length). Numerical solutions of Eq. 16 for desorption and absorption. Curve 2: u_o = 1.0 g/g, u_i = 0.0 g/g (desorption). Curve 3: u_o = 3.0 g/g, u_i = 0.0 g/g (desorption). Curve 4: u_o = 0.0 g/g, u_i = 1.0 g/g (absorption). Curve 5: u_o = 0.0 g/g, u_i = 3.0 g/g (absorption). True density of dry solids in all cases: ρ_s = 1.2 g/cm³. Density of water: ρ_w = 1.0 g/cm³.

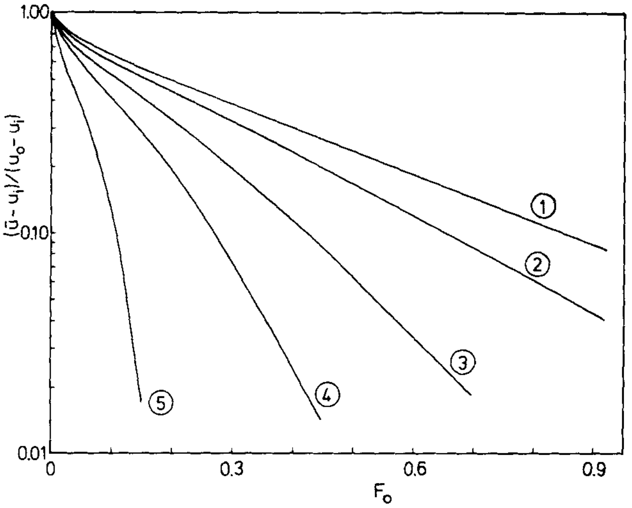

Figure 2. Dimensionless moisture content vs. Fourier number based on the initial half-thickness of the slab for desorption. Curve 1: Equation 3 (no shrinkage, R_o as characteristic length). Numerical solutions of Eq. 16 for desorption. Curve 2: $u_o = 0.15$ g/g, $u_i = 0.0$ g/g ($v_i = 1/1.18$). Curve 3: $u_o = 0.5$ g/g, $u_i = 0.0$ g/g ($v_i = 1/1.6$). Curve 4: $u_o = 1.0$ g/g, $u_i = 0.0$ g/g ($v_i = 1/2.2$). Curve 5: $u_o = 3.0$ g/g, $u_i = 0.0$ g/g ($v_i = 1/4.6$). True density of dry solid used in all cases: $\rho_s = 1.2$ g/cm^3. Density of water: $\rho_w = 1$ g/cm^3. Note: The solutions are valid for other combinations of variables u_o, u_i, ρ_s, and ρ_w that give the corresponding values of the variable v_i given for each curve.

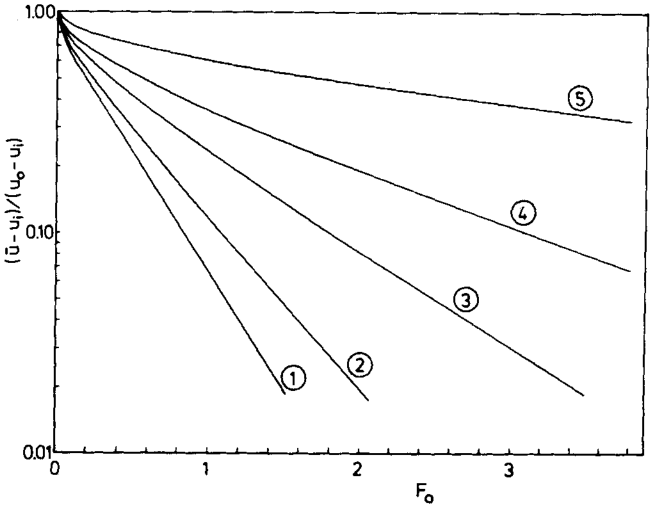

Figure 3. Dimensionless moisture content vs. Fourier number based on the initial half-thickness of the slab for absorption. Curve 1: Equation 3 (no shrinkage, R_o as characteristic length). Numerical solution of Eq. 16 for absorption. Curve 2: $u_o = 0.0$ g/g, $u_i = 0.15$ g/g ($v_i = 1.18$). Curve 3: $u_o = 0.0$ g/g, $u_i = 0.5$ g/g ($v_i = 1.6$). Curve 4: $u_o = 0.0$ g/g, $u_i = 1.0$ g/g ($v_i = 2.2$). Curve 5: $u_o = 0.0$ g/g, $u_i = 3.0$ g/g ($v_i = 4.6$). True density of dry solid used in all cases: $\rho_s = 1.2$ g/cm^3. Density of water: $\rho_w = 1$ g/cm^3. Note: The solutions are valid for other combinations of variables u_o, u_i, ρ_s, and ρ_w that give the corresponding values of the variable v_i given for each curve.

NUMERICAL SOLUTION OF EQ. 16

Equation 16 was solved numerically by two different methods: an explicit method with forward differences and the three-level implicit method proposed by Lees (1959). Both methods give results that agree satisfactorily. In Figures 1, 2, and 3 are given the numerical results as a function of either the Fourier number based on the dry half-thickness or the Fourier number based on the initial half-thickness.

For the purpose of comparison Figure 1 shows the results predicted by Eq. 3 using as the significant length the dry half-thickness R_s. Figures 2 and 3 show the predictions of Eq. 3 using as the significant length the initial half-thickness R_o. It can be observed that both solutions differ significantly from those obtained by Eq. 3, particularly in cases where the initial moisture content of the slab is high.

It is worth mentioning that Figures 2 and 3 are completely general; in these figures the dimensionless moisture content vs. Fourier number (based on the initial half-thickness) are plotted, the ratio R_i/R_o being the significant parameter. However, Figure 1 is only valid for the particular values of the physical parameters used in the numerical solutions (ρ_s, ρ_w, u_o, u_i). This is because it is not possible to obtain a general monoparametric solution in terms of Fourier number based on the dry half-thickness. Accordingly, Figure 1 was included only to show the different behavior of a slab with the same dry half-thickness and subjected to symmetrical conditions of absorption and desorption.

It can be observed from Figures 1, 2, and 3 that a false concentration dependence of the diffusion coefficient appears where experimental drying runs are performed with different initial moisture content. This is so because the usual procedure is to obtain the diffusion coefficient from the slope of the plot of moisture content vs. time in semilogarithmic coordinates, and, as can be seen from Figures 1, 2, and 3, the slope is a function of the initial moisture content.

Besides, if experimental drying runs are performed at different temperatures in order to obtain the activation energy of the diffusion process, false results are obtained from Eq. 3 when the experimental runs are conducted with different initial moisture contents. This fact provides an explanation for the considerable standard deviation in the calculation of the activation energies (Jason, 1958; Vaccarezza et al., 1974).

Figure 1 shows that the kinetic curves for absorption are different from those of desorption. This explains the different diffusion coefficients for absorption and desorption pointed out by King (1968). Finally, at low moisture contents the solid can become a rigid instead of a shrinking body, and a change in the drying kinetics might be expected. This hypothesis, if experimentally verified, would provide an alternative explanation for the appearance of the second falling drying rate period.

NOTATION

A = component that is diffusing (water)
C_A^B = concentration of component A by unit volume of dry solid B, g/cm^3
D = mutual diffusion coefficient
D_A^B = diffusion coefficient for a frame fixed to the inert solid B
D_{eff} = effective diffusion coefficient
F_o = $D\theta/R_o^2$, Fourier number based on the initial half-thickness of the slab
F_s = $D\theta/R_s^2$, Fourier number based on the dry half-thickness of the slab
R = half-thickness of the slab
R_o = initial half-thickness of the slab

R_i = half-thickness of the slab with an equilibrium moisture content
R_s = half-thickness of the slab totally dried
u = moisture content of the solid, dry basis
$\bar{u}$ = mean moisture content, dry basis
v = $(1 + \rho_s u/\rho_w)/(1 + \rho_s u_o/\rho_w)$
x = coordinate along the diffusion path
z = ξ_B/R_s, dimensionless coordinate along the diffusion path

Greek Letters

ρ_A = mass concentration of component A by unit volume
ρ_B = mass concentration of solid B by unit volume
ρ_s = mass density of fully dried solid (squeletal density)
ρ_w = mass density of diffusant (water)
ξ_B = volume of solid B by unit transversal area
θ = time

Subscripts

o = initial condition
i = interface condition (equilibrium)

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